

PSIG 1302

## **Mathematical modeling of wave processes in leak detection systems (LDS)**

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### **ABSTRACT**

LDS, detecting pressure drop surge seem to be the most effective for the leak detection in pipelines. In fact, such systems have very high sensitivity. Despite that there are false leak detections, when disturbances in the pipeline are not related to leakage (such disturbances might be caused by changes in the frequency of pumps, pressure regulating systems work, etc.) Present article addresses the problem of minimizing false leak detections, as well as increasing sensitivity of algorithms, based on the pressure drop surge.

### **INTRODUCTION**

With the continuous growth of the total length and the aging of pipelines that have been operated for many decades, the risk of leaks and major technogenic accidents increases. To reduce the damage caused by such accidents, it is necessary to monitor a number of parameters in the pipeline system.

Despite the large variety of leak detection systems, a significant amount of theoretical papers, practical and engineering proposals in this area, the problem of online leak detection (both large and small) is so sophisticated, that the universal solution, which is suitable for a full range of conditions on working pipelines is still not found. That is why the creation of new methods and tools for the detection of oil leakages remains relevant.

To evaluate advantages and disadvantages of separate methods of leak detection one should take into account various circumstances. LDS characteristics depend on a number of

factors: geometric parameters of a pipeline, properties of a transported fluid, technological parameters of a pumping mode, technological features of the equipment, etc. Specific set of affecting parameters (more precisely a share of impact of each parameter) depends on the selected method of leak detection. Besides, some methods can detect only the fact of leakage, while others allow identification the exact leak location.

The basic requirements for control systems are:

- high sensitivity ;
- accuracy of the leak location;
- absence of false detections;
- high degree of reliability;
- profitability.

Many methods that use a variety of principles (even considering biolocation) exist, but around the world, the most common are methods, based on the analysis of hydrodynamic processes.

The most popular are pressure surge leak detection systems, based on the detection of the pressure downsurge, occurring in the pipeline during the leak lifecycle. In fact, such systems have very high sensitivity. They are capable to detect the leak from an aperture of 4-5 mm (1.9-2.4 li) with pressure drop of 0,02 atm(bar).

Such systems detect the location of the leak with accuracy up to 30-50 m (yd), and time of response is 20-30 sec.

This work offers an addition to this method. It includes mathematical model that describes the transient processes of fluid motion in a pipeline. The idea is to extract the disturbance, associated with the leakage with the use of the flow simulation and real measurements of pressures in the pipeline.

The obtained results allow timely and accurately determine the location of leaks (including small leaks with flow rate up to

0.5% of the flow rate in a pipeline) and illegal connections and, most important, to avoid false leak detections when the non-leak pressure disturbances appear in the pipeline.

## PRESSURE DOWNSURGE METHOD

The method involves the detection of pressure downsurge that occurs during the formation of the perforation in a pipe. Downsurge wave propagates from the leak in both directions and can be detected with high precision sensors along the pipeline.

Let us consider the pressure downsurge method in details. The first question to discuss is: "What is a pressure wave?"

Figure 1 shows plots of pressures from two sensors about 200 meters (218 yards) apart. At some moment the valve, located at a distance of 7.4 km (4.6 mi) away from the closest pressure sensor, sharply opens. The discharge of oil is produced in the outer tank. It results in a pressure drop that propagates along a pipe. This drop was detected at time  $t = 65$  sec by the closest sensor from the point of discharge. About 170 msec after the first detection, second sensor detected the pressure drop. After 3-4 seconds, the valve was closed, the oil discharge from the pipeline stopped and pressure in the section of oil discharge risen sharply. This pressure upsurge was also detected by these pressure sensors. Further LDS-algorithm, based on the detection of a pressure drop in a pipeline is called « the pressure wave algorithm».

LDS "pressure wave algorithms" can detect the fact of leak, as well as localize it.

They usually consist of the following stages:

- 1) Detect pressure drop on the controlpoint (hereinafter - CP), which conduct pressure measurements, in front of leak point and on the CP after leak point.
- 2) Calculate the difference of absolute times of the pressure drop detection on both CP.
- 3) Calculate the coordinates of the leakage.

For implementation of this algorithm, a special program is functioning at the middle level (in the controller installed at CP) or on the upper level of the telemechanics system. Pressures used in the algorithm must be measured periodically with a frequency of 20-100Hz. These measurements are provided by special LDS-controllers. To detect the pressure drop digital filters and correlation analysis are used. If the pressure drop is detected, a message is formed in the software of the upper level, which makes the final data analysis.

For leak detection is necessary to observe the pressure drop at least on two local LDS controllers. The sensitivity of the

method depends on the dynamic characteristics of sensors, level of the hydrodynamic noise in a pipeline, accuracy of time synchronization, information processing performance of the controller and accuracy of the measurements or calculations of the acoustic sound velocity.

A significant impact on the effectiveness of the method is made by the presence of gas bubbles in the pipeline, which absorb waves of pressure disturbance. This has a negative influence on the performance of the method.

## MATHEMATICAL MODELING IN LDS.

Due to the intensive development of pipeline transport, new approaches are applied in LDS - creating methods and algorithms based on mathematical modeling of oil pumping. This trend is related to the increase of complexity of technical equipment and automatic control algorithms in the pipeline, and more strict security requirements of oil transport .

Old techniques quickly lose their effectiveness because of their inability to take into account the "external" disturbances - the impact that exerted on the pumping processes by using technical equipment (for example, pump control systems), pigs, as well as special drag reducing additives. All this leads to a huge number of false detections and completely discredits the system.

Furthermore, the knowledge and ingenuity of criminals involved in oil theft increase over time. The theft commonly occurs during the transient processes in pipelines, for example at the start or stop of the pumping. Old methods in such transient processes can detect only very large leaks.

In most cases such leaks can not be detected at all, because criminals maintain the leakage flow rate to be low, so the sensitivity of the LDS is not high enough to detect the leak.

Applying of the principles of mathematical modeling in LDS avoids most of false detections in case if the model with good accuracy considers all "external" sources of disturbances. In addition, usage of the mathematical model of hydraulic flow allows simulating pumping processes with a certain degree of accuracy (depending on the numerical method) and based on the analysis of mathematical solution and real measurements to judge about presence of the leakage in a pipeline and to increase the sensitivity of system.

This article offers a solution of the problem of minimizing the number of false alarms, caused by "external" disturbances in steady-state pumping. The idea of solution is based on the construction of a mathematical model, which considers all sources of disturbances not related to leakage. To take into account such disturbances, pressure measurements on the next CP from the disturbance source are used as the boundary conditions. As additional parameters for calculation measurements of flow volume, density, viscosity and values of

the inner diameter in the pipeline are considered. The model calculates pressure distribution along the pipeline. Most often the sources of "external" disturbances are different technological switchings on the pumping stations and in tank farms. Therefore, the calculation of the pressure distribution is performed along the linear part of the pipeline between two pumping stations. Boundary conditions are considered to be equal to pressure measurements before and after the technological binding of the stations.

To determine the presence or absence of leak in a pipeline section, the calculated (without the effect of leak) and the measured pressure distributions are compared. In case of a leakage a significant discrepancy between the measured and calculated pressures appears. Moreover, this discrepancy shows a "clean" leak without disturbances from other sources. Thus this method allows to "filter out" the leak. As a result, the error in determining the localization of leakage is reduced significantly and it becomes possible to increase system sensitivity greatly.

## MATHEMATICAL MODEL

### 1. Method of Characteristics

The most efficient numerical method for unsteady fluid motion in pipelines is the method of characteristics. This paragraph contains brief description of this method.

Consider a linear, in general case, nonhomogeneous system of equations:

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{A} \frac{\partial \mathbf{U}}{\partial x} = \mathbf{b}, \quad (1)$$

$$\mathbf{L} \cdot \mathbf{A} = \kappa \cdot \mathbf{L}, \quad (2)$$

where  $t$  – time;

$x$  – coordinate for length;

$\mathbf{U}$  – unknown vector-function of order  $n$ ;

$\mathbf{A}$  – matrix of order  $n$ ;

$\mathbf{b}$  – vector of the right side;

$\mathbf{L}$  – left eigenvector of the matrix  $\mathbf{A}$

$\kappa$  – eigenvalue.

The system (1), (2) is called hyperbolic if the eigenvalues  $\kappa_1, \kappa_2, \dots, \kappa_n$  of the matrix  $\mathbf{A}$  are real and different from

each other.

Let us make an assumption that the system (1), (2) is hyperbolic. Multiply it on the left of the  $j$ -th eigenvector corresponding to the value  $\kappa_j$  and consider the equation (2).

$$L^j \frac{\partial \mathbf{U}}{\partial t} + L^j \mathbf{A} \frac{\partial \mathbf{U}}{\partial x} = L^j \cdot \left( \frac{\partial \mathbf{U}}{\partial t} + \kappa_j \frac{\partial \mathbf{U}}{\partial x} \right) = L^j \mathbf{b}, \quad (3)$$

where  $j = 1, \dots, n$

The expression in brackets is the derivative along the line given by the equation:

$$\frac{dx}{dt} = \kappa_j, \quad (4)$$

Reflect this fact with the following notation:

$$\frac{\partial}{\partial t} + \kappa_j \frac{\partial}{\partial x} = \left( \frac{d}{dt} \right)_j, \quad (5)$$

Curves defined by equations (4) are the characteristics of equation (1). Equation (3) considering (5) can be written as:

$$L^j \left( \frac{d\mathbf{U}}{dt} \right)_j = L^j \mathbf{b}, \quad (6)$$

Relations (6) are called the characteristic form of system (1), (2). Their property is that in each equation (6) the differentiation is carried along only one characteristic.

### 2. Application of the method of characteristics for calculating weakly compressible fluid flow.

Now we can apply our results to the integration of the equations of continuity and momentum conservation for the flow of a viscous compressible fluid in a cylindrical tube:

$$\frac{\partial p}{\partial t} + \rho c^2 \frac{\partial u}{\partial x} = 0, \quad (7)$$

$$\frac{\partial u}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} + \lambda \frac{u \cdot |u|}{2 \cdot D} + g \sin \gamma = 0, \quad (8)$$

where  $P$  – pressure;

$\lambda$  - flow friction characteristic;

$\gamma$  - angle to the horizontal line;

$D$  - inner diameter of the pipeline.

This system of equations is hyperbolic with any  $(x, t)$  and the characteristic equations (7) and (8) can be written as

$$\left(\frac{u}{c} \ll 1\right):$$

$$dx - c \cdot dt = 0, \quad (9)$$

$$dx + c \cdot dt = 0, \quad (10)$$

And relations to these characteristics, respectively:

$$dp + \rho \cdot c \cdot du + \rho \cdot c \left( \lambda \frac{|u|}{2 \cdot D} u + g \sin \gamma \right) \cdot dt = 0, \quad (11)$$

$$dp - \rho \cdot c \cdot du - \rho \cdot c \left( \lambda \frac{|u|}{2 \cdot D} u + g \sin \gamma \right) \cdot dt = 0; \quad (12)$$

A numerical method is constructed as follows - for each distributed element, we introduce a grid:

$$\Omega = \{ (m \cdot \Delta x, n \tau), n = 0 \dots N, m = 0 \dots M \}, \quad (13)$$

where  $\Delta x$  and  $\tau$  - steps in length and time accordingly.

Grid is constructed in the way that insures that the Courant condition is satisfied. For this system, it would be:

$$\frac{c \tau}{\Delta x} < 1, \quad (14)$$

Frequency of the data measurements defines the time step of integration. Knowing the time step and using (14), step in length can be defined.

Characteristics of (11) and (12) have the form of straight lines as show in Figure 2

Knowing the pressure at all points of the grid on the n-th time step, we find the pressure values at points A and F, by interpolating values from the neighboring nodes

$p_{m-1}^n, p_m^n, p_{m+1}^n$  as follows:

$$p_A = \frac{c \tau}{\Delta x} p_{m-1} + \left(1 - \frac{c \tau}{\Delta x}\right) p_m, \quad (15)$$

$$p_F = \frac{c \tau}{\Delta x} p_{m+1} + \left(1 - \frac{c \tau}{\Delta x}\right) p_m, \quad (16)$$

Similarly, we find the values of the velocity at points A and F:

$$u_A = \frac{c \tau}{\Delta x} u_{m-1} + \left(1 - \frac{c \tau}{\Delta x}\right) u_m, \quad (17)$$

$$u_F = \frac{c \tau}{\Delta x} u_{m+1} + \left(1 - \frac{c \tau}{\Delta x}\right) u_m, \quad (18)$$

The scheme of predictor-corrector method is used for finding the parameters of the flow at the point B.

Predictor. To write the difference analogue of (11) and (12) on the characteristics of the AB and FB:

AB:

$$\bar{p}_B - p_A + \rho_A \cdot c \tau \left( \frac{\lambda_A u_A |u_A|}{2D} + g \sin \gamma_A \right) + \rho_A c (\bar{u}_B - u_A) = 0, \quad (19)$$

FB:

$$\bar{p}_B - p_F - \rho_F \cdot c \tau \left( \frac{\lambda_F u_F |u_F|}{2D} + g \sin \gamma_F \right) - \rho_F c (\bar{u}_B - u_F) = 0, \quad (20)$$

The bar over the parameters of the point B means that this is not a final solution at this point, but only the value given to the predictor. Density at each particular point is recalculated after pressure by Hooke's law:

$$\rho = \rho_0 \left(1 + \frac{P - P_0}{K_g}\right), \quad (21)$$

where  $K_g$  - bulk modulus of the fluid;

$\rho$  - density;

$P_0$  - normal pressure;

$\rho_0$  - density under pressure  $P_0$ .

Solving (19) and (20), we obtain:

$$\begin{aligned} \bar{u}_B = & \frac{\rho_A u_A + \rho_F u_F}{\rho_A + \rho_F} + \frac{p_A - p_F}{c(\rho_A + \rho_F)} + \frac{\tau}{\rho_A + \rho_F} \left( \frac{\lambda_A \rho_A u_A |u_A| + \lambda_F \rho_F u_F |u_F|}{2D} + \right. \\ & \left. + \rho_A g \sin \gamma_A + \rho_F g \sin \gamma_F \right), \end{aligned} \quad (22)$$

$$\bar{p}_B = p_A - \rho_A \cdot c \tau \left( \frac{\lambda_A u_A |u_A|}{2D} + g \sin \gamma_A \right) - \rho_A c (\bar{u}_B - u_A), \quad (23)$$

Corrector. Interpolate the values at the points W and E with the values at the points A, F, and the values at the point B, obtained from (22) and (23):

$$p_W = \frac{p_A + \bar{p}_B}{2}, u_W = \frac{u_A + \bar{u}_B}{2}, \quad (24)$$

$$p_E = \frac{p_F + \bar{p}_B}{2}, u_E = \frac{u_F + \bar{u}_B}{2}, \quad (25)$$

Rewrite (19) and (20) as follows:

AB:

$$p_B - p_A + \rho_W \cdot c \tau \left( \frac{\lambda_W u_W |u_W|}{2D} + g \sin \gamma_W \right) + \rho_W c (u_B - u_A) = 0, \quad (26)$$

FB:

$$p_B - p_F - \rho_E \cdot c \tau \left( \frac{\lambda_E u_E |u_E|}{2D} + g \sin \gamma_E \right) - \rho_E c (u_B - u_A) = 0, \quad (27)$$

The final values of pressure and velocity at point B are obtained from the following formulas:

$$u_B = \frac{\rho_W u_A + \rho_E u_F}{\rho_W + \rho_E} + \frac{p_W - p_E}{c(\rho_W + \rho_E)} + \frac{\tau}{\rho_W + \rho_E} \left( \frac{\lambda_W \rho_W u_W |u_W| + \lambda_E \rho_E u_E |u_E|}{2D} + \rho_W g \sin \gamma_W + \rho_E g \sin \gamma_E \right), \quad (28)$$

$$p_B = p_A - \rho_W \cdot c \tau \left( \frac{\lambda_W u_W |u_W|}{2D} + g \sin \gamma_W \right) - \rho_W c (u_B - u_A). \quad (29)$$

Boundary conditions. Because in this system the two eigenvalues have different signs, boundary conditions should be set as follows: one from the left side of tube ( $n = 0$ ) and one from the right ( $n = N$ ). Then at the end points the system, which consists of a single boundary condition and ratio brought in by characteristic at this point, must be solved.

Consider for example the right border with the given pressure on it. Characteristic AB (see Fig. 3) brings the ratio (11), a difference analogue of which has the form (19).

Substituting in (19) instead of  $\bar{p}_B$  the pressure value at the right boundary we obtain an expression for the predictor of velocity:

$$\bar{u}_B = u_A + \frac{p_A - p_B}{\rho_A c} - \tau \left( \frac{\lambda_A u_A |u_A|}{2D} + g \sin \gamma_A \right), \quad (30)$$

Using this value with formulas (24-25) we find the values at the point W, and then, using (28-29) we find the corrected velocity  $u_B$ :

$$u_B = u_A + \frac{p_A - p_B}{\rho_W c} - \tau \left( \frac{\lambda_W u_W |u_W|}{2D} + g \sin \gamma_W \right), \quad (31)$$

Solution at the left boundary is similar to that described the solution on the right boundary.

## IMPACT OF INPUT PARAMETERS

The mathematical model takes measured pressure, diameters of the pipeline, acoustic speed in the sections between pressure sensors, average values for viscosity and density of oil as input parameters. The influence of the input parameters on the accuracy of the calculation is obvious.

Quality of the pressure measurements is highly important for the algorithm. The method of leak detection for pressure downsurge is primarily based on the detection of the front of the pressure downsurge wave. Due to this fact the calculation accuracy of leak location depends on the reliability of time when pressure downsurge is detected on the different pressure sensors and, therefore, on the frequency of the measurements. On different sensors the disturbance of leak comes at different times depending on the distance from sensors to the leak. Therefore, to determine the leak location, two sensors are selected based on time of pressure downsurge detection. If the pressure downsurge threshold on the sensor  $k$  is breached at  $t_K$ , and on the sensor  $k + 1$  at  $t_{K+1}$ , distance  $X_K$  from the sensor  $k$  to the leak is given by

$$X_K = \frac{L}{2} + \frac{c(t_K - t_{K+1})}{2} \quad (32)$$

where  $c$  - acoustic speed,  $L$  - length of the section.

To minimize the error in determining leak location to tens of meters, pressure measurements with a frequency of at least 50-100 Hz are required.

Analysis of other input parameters shows that acoustic speed has a great influence on the convergence of the model. In case of incorrect value of acoustic speed, a shift in time between the measured and calculated signals appears, as shown in Fig.4 - 7. This leads to an increase in the difference between signals, which can cause false leak detections. Acoustic speed depends on the properties of the transported fluid (temperature, density, etc.) and the properties of the material

and wall thickness of the pipeline, which is not quite uniform. Along the pipeline acoustic speed can have different value in different sections. Because output of the algorithm depends on the convergence of calculated and measured signal at CP, acoustic speed is identified between two adjacent CP.

Theoretically it is possible to calculate the acoustic speed with the formula of N.E Zhukovsky:

$$c = \frac{1}{\sqrt{\frac{\rho}{K} + \frac{\rho d}{E\delta}}} \quad (33)$$

where  $K$  - elastic modulus of fluid,  $\rho$  - density,  $d$  - diameter;  $E$  - Young's modulus of the pipe material,  $\delta$  - wall thickness,

but usually it is difficult or impossible to determine all of the parameters in the formula (33).

Practically correlation method is used to calculate the acoustic speed. In real operational systems (with data frequency of 50 Hz) accuracy of determining acoustic speed is about 1% (10 - 15 m/sec (10-15 ya/sec)).

## RESULTS OF TESTING

Proposed approach has been successfully applied in certain LDS on operational pipelines, serviced by our company. This paragraph contains some testing results of the existing implementation of the approach.

For convenience, all results are shown in pressure deviations from the mean value.

During the test, there were artificial leakages at 35.5 km (22.059mi) mark, providing short time discharge of oil (about 20seconds) to the external tank. Drain oil flow does not exceed 0.5% of flow rate in the pipeline. Flow rate is 11 m<sup>3</sup>/h (2420gal-uk/h) in inner diameter of 710 mm (28 inch).

Recorded measurements during this test were processed by mathematical model. Figures 8 - 10 show measured and calculated pressures and the difference between them.

Besides leaks related pressure downsurge, there are a number

of pressure disturbances, caused by the work of pressure control system on the pumping station. Such pressure disturbances can extend far enough along the pipeline and LDS, operating without the use of mathematical model, can trigger false leak detection on them. As the graphs show, the simulated pressure curve has good convergence to the real measurements. Pressure surges related to leakage are not filtered out, and can be easily distinguished.

In practice LDS systems often use different correlation algorithms to emphasize more clearly leak disturbance and smoothen hydraulic noise. We apply one of such methods to get the results. Figure 11 shows values of the correlation function applied to difference between measured and calculated curves. Possible threshold for leak detection is equal to 0.02. For comparison, Figure 12 shows the result without the use of mathematical model. It shows values of the correlation function which was applied to measured pressure. The graph shows that detection of this leak in second case failed.

Part of the test results is shown in Table 1. As one can see from the results, method can detect leaks at rate of 0.25% of the overall flow rate. It also gives good accuracy in determining the leak location - the error on average is about 200m (200 ya) (appresiable error ranges from 400 to 800 m (400-900 ya), depending on the leakage parameters). Response time is about 3 minutes. No false detection has been triggered by LDS.

## CONCLUSIONS

As you can see at the moment leak detection systems are facing a number of problems. Various improvements of technical equipment of the pipeline, complexity and automating processes of pumping leads to a huge number of artificial disturbances in the pipeline (even in steady-state pumping), which LDS must filter out. It becomes necessary to include modeling of fluid flow to solve these problems.

This article describes the approach, suitable for improvement of leak detection method based on the pressure drop. It offers a solution to this problem, involving mathematical model. As one can see from test data, implementation of this approach has shown good accuracy of leak detection and absence of false detections on real operational pipelines.

## TABLES

Characteristics of oil discharge							Reaction LDS			
№	Time of the opening hh:mm:ss	Duration of the opening sec	Duration of the leak hh:mm:ss	The flow rate of oil flowmeter			Detected leak location km(mi)	Error of leak location ±km(mi)	Calculated flow of leak m <sup>3</sup> /h (gal-uk/h)	Response time LDS mm:ss
				% of the maximum performance	m <sup>3</sup> /h (gal-uk/h)	Real oil volume of leak m <sup>3</sup> (gal-uk/h)				
1	8:49:30	00:01	0:00:20	0.24	11 (2420)	0.051	35.488 (22.051)	0.012 (0.008)	12.9 (2838)	02:33
2	8:58:40	00:03	0:00:20	0.24	12 (2640)	0.051	35.316 (21.944)	0.184 (0.115)	12.3 (2706)	02:25
3	9:06:15	00:10	0:00:20	0.24	12 (2640)	0.051	34.716 (21.572)	0.216 (0.487)	12.9 (2838)	02:43
4	9:21:30	00:10	0:00:20	0.24	11 (2420)	0.051	35.485 (22.049)	0.015 (0.010)	12.3 (2706)	02:26

**Table 1 - Detection results for the artificial leak.**

## FIGURES

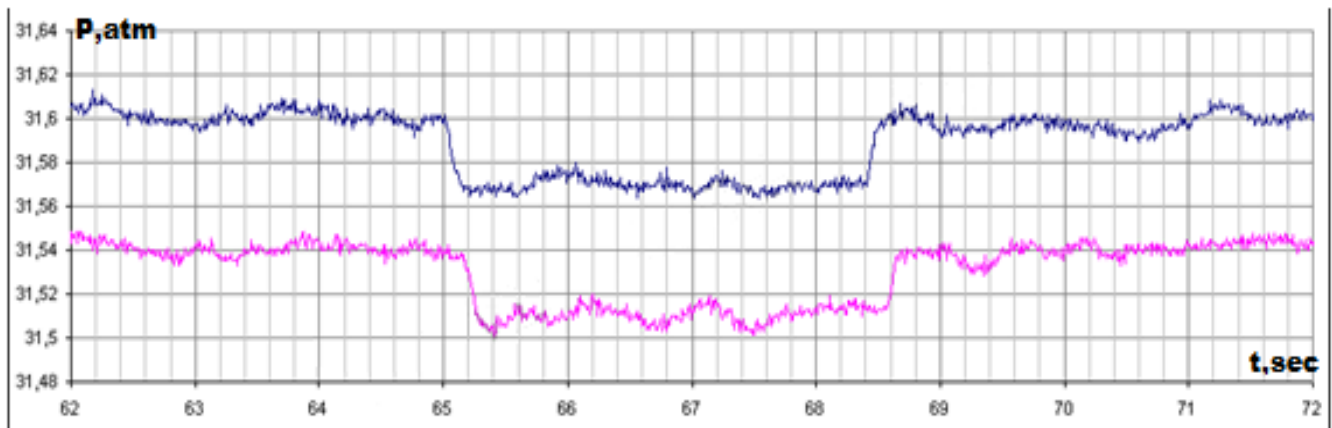


Figure 1 - "pressure downsurge wave" caused by leakage

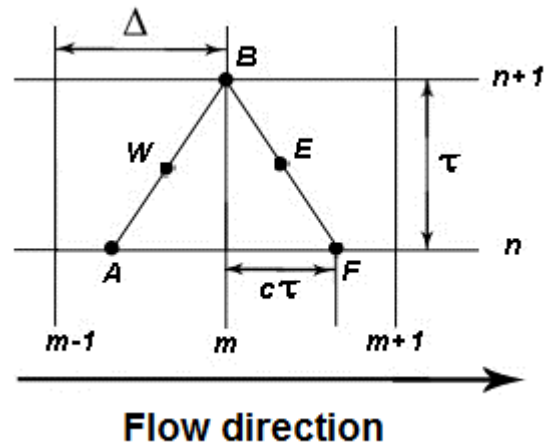


Figure 2 – Part of the characteristics grid.

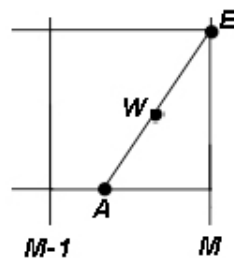


Figure 3 - Boundary conditions.



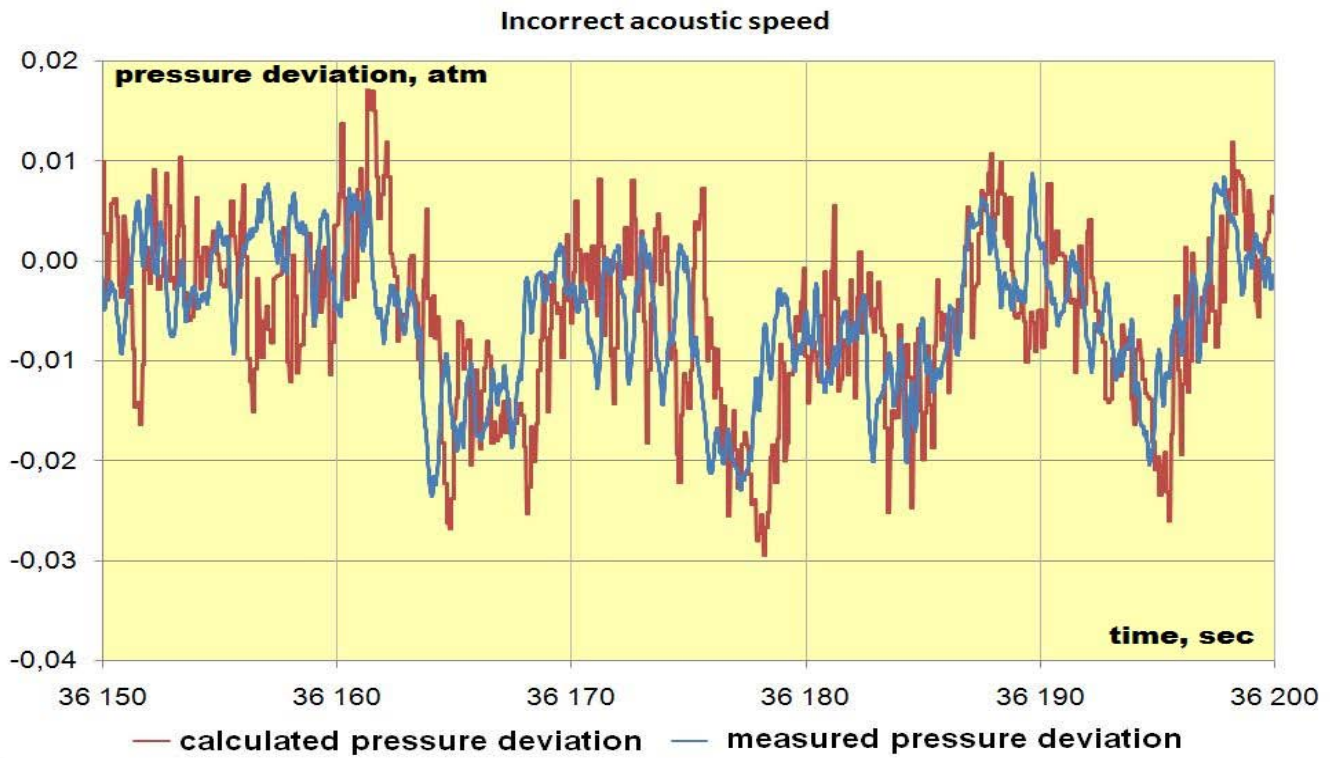


Figure 4 – Measured (blue) and calculated (red) pressure curves. Acoustic speed - incorrectly defined

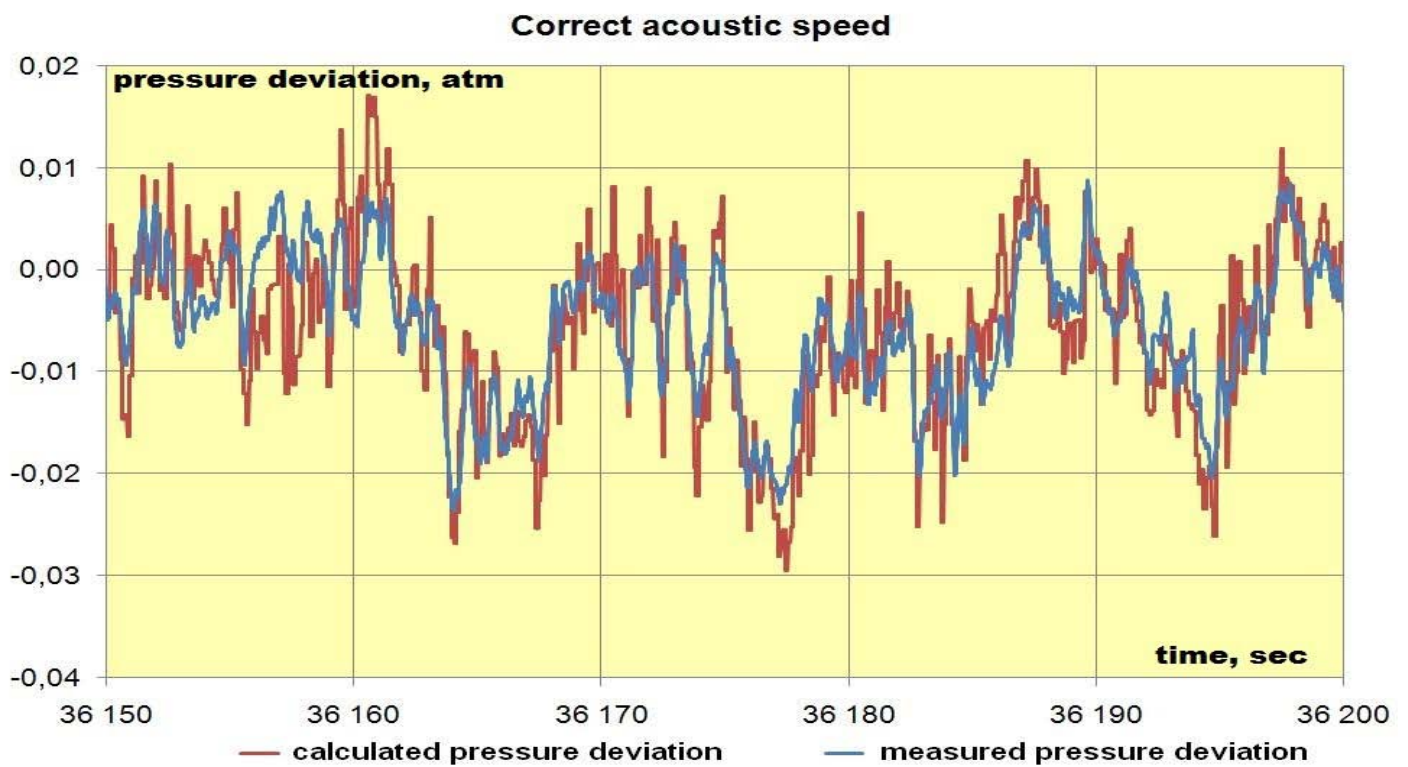


Figure 5 – Measured (blue) and calculated (red) pressure curves. Acoustic speed - correctly defined

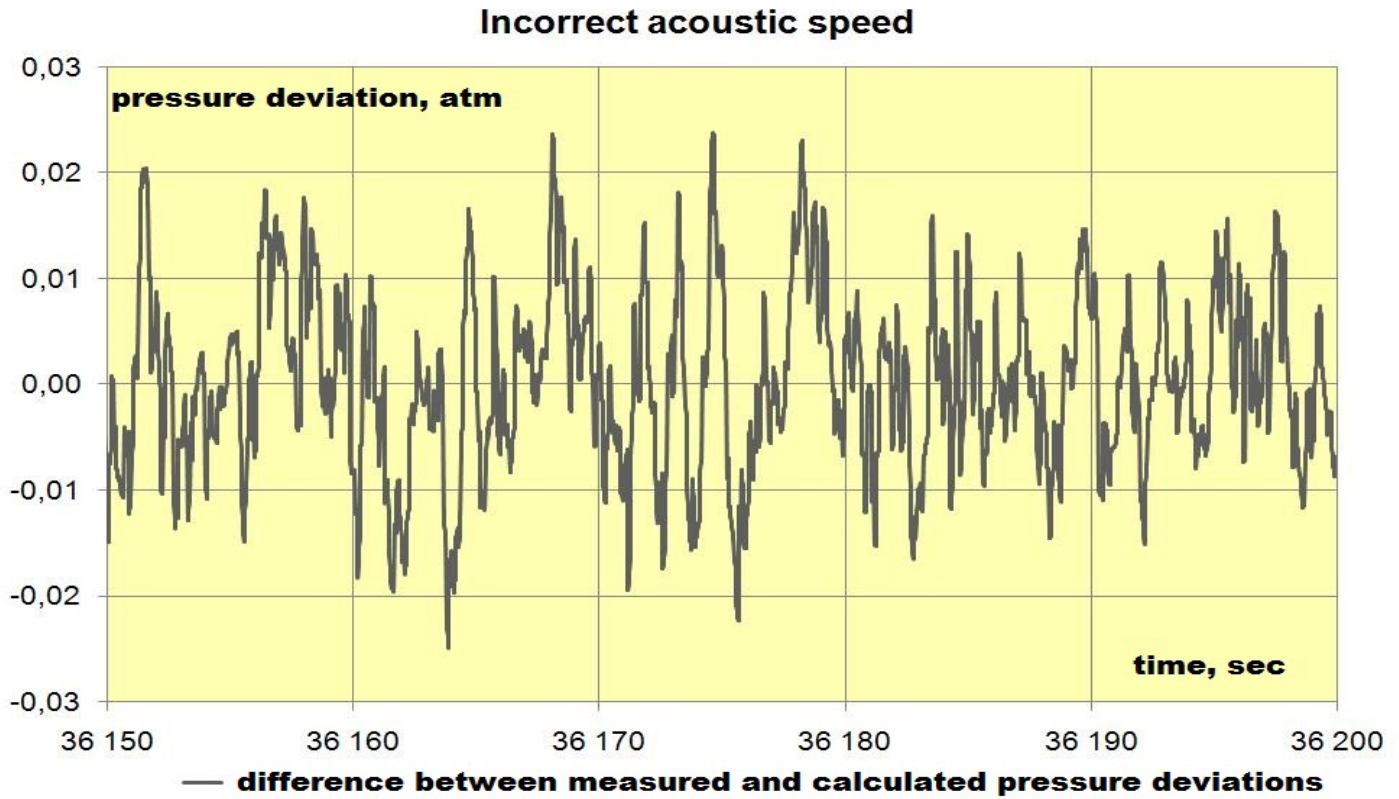


Figure 6 – Difference between calculated and measured pressure curves for incorrect acoustic speed.

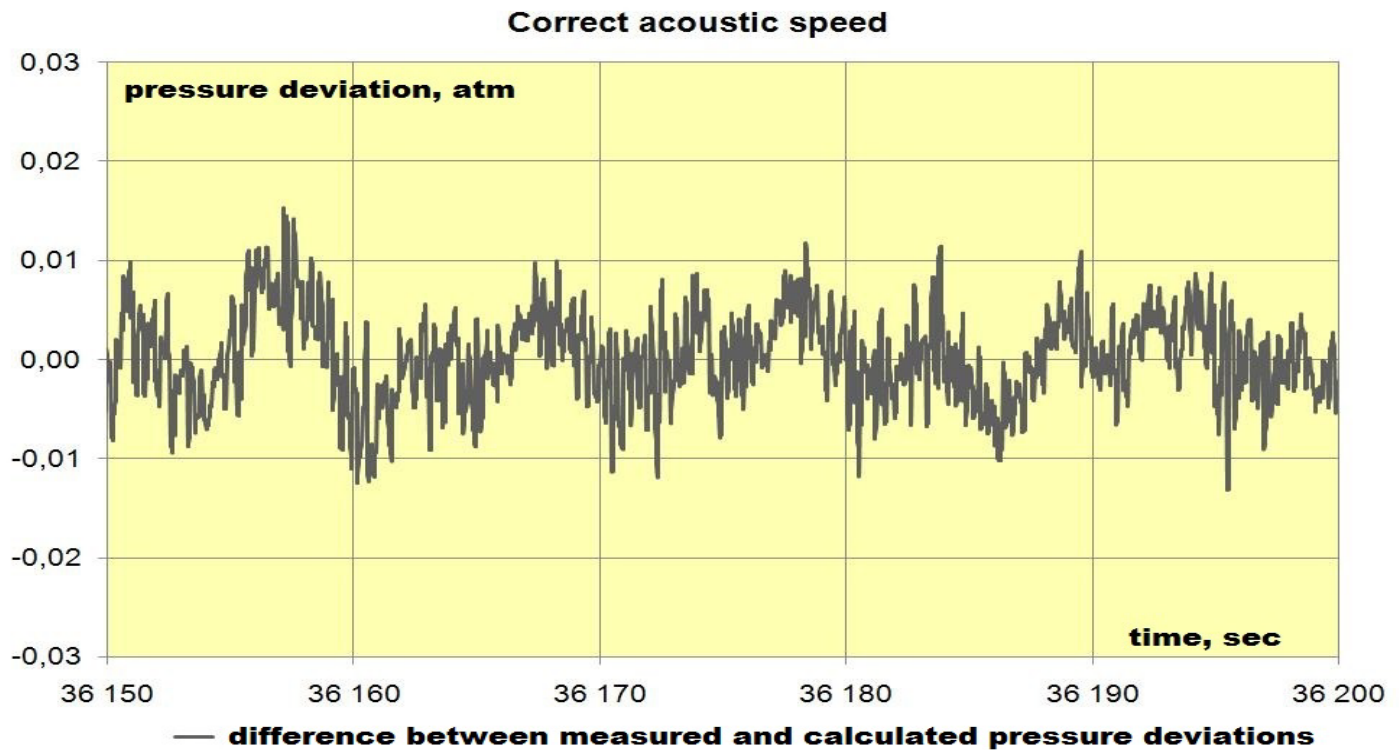


Figure 7 – Difference between calculated and measured pressure curves for correct acoustic speed.

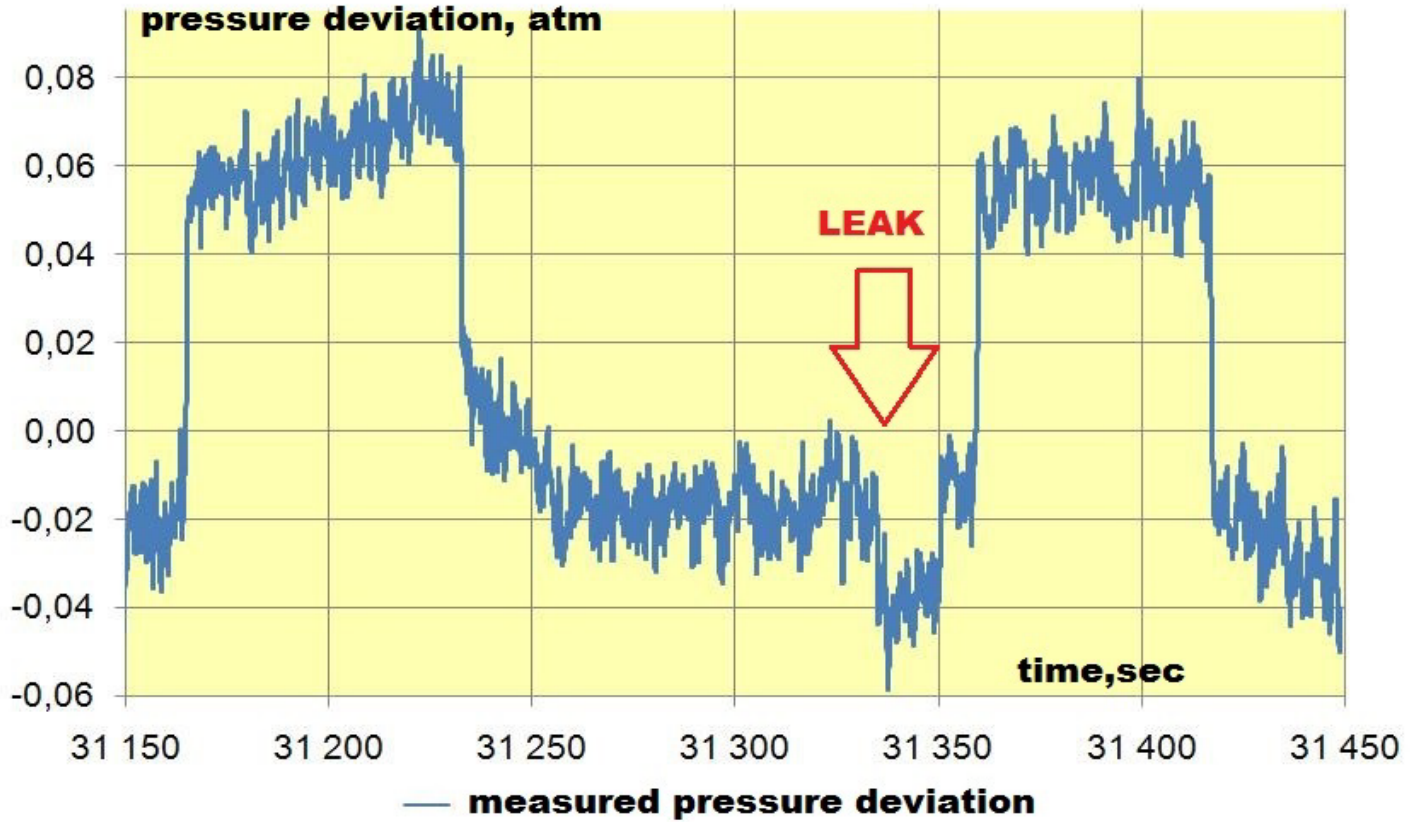


Figure 8 – Oil discharge at 31335 sec with duration of 20sec. Measured pressure curve.

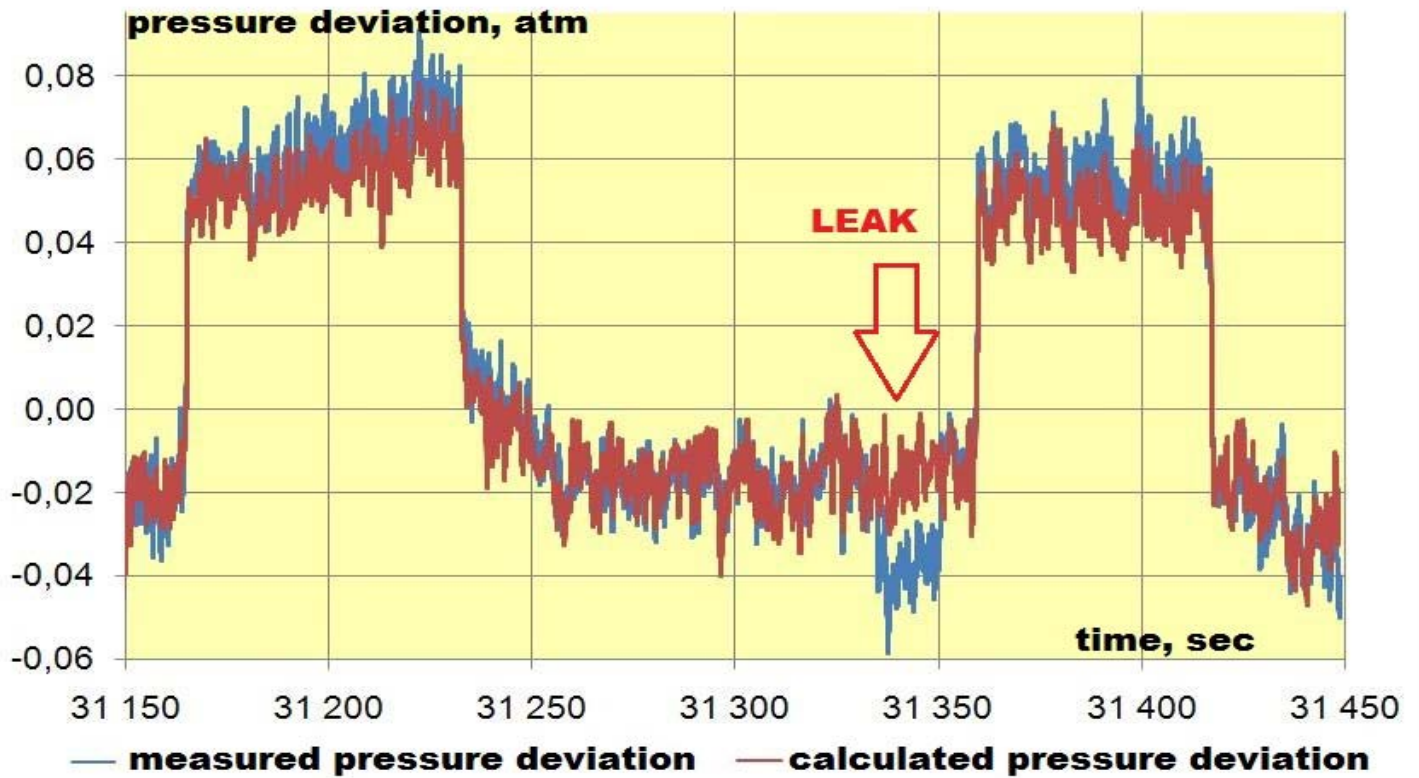


Figure 9 – Oil discharge at 31335 sec with duration of 20sec. Measured (blue) and calculated (red) pressure curves.

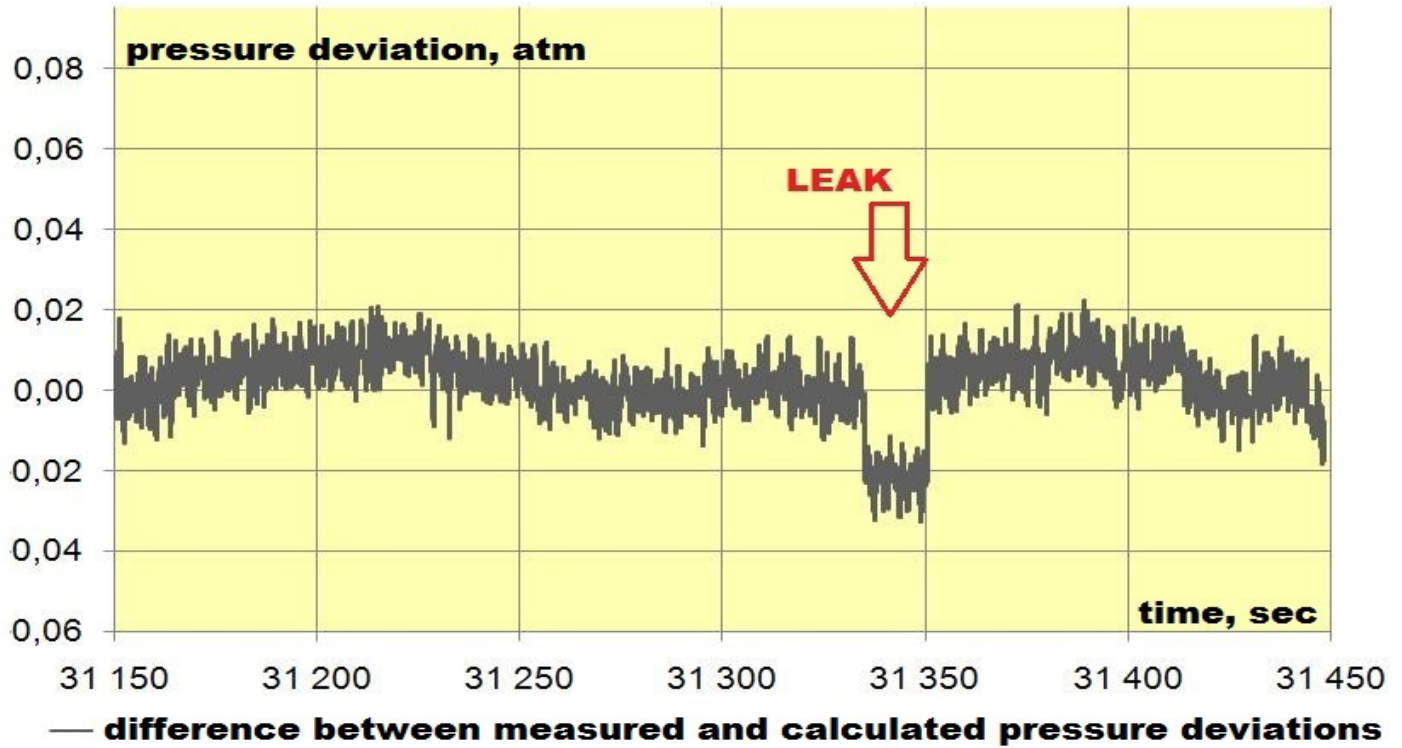


Figure 10 – Oil discharge at 31335 sec with duration of 20sec. Difference between measured and calculated pressure curves.

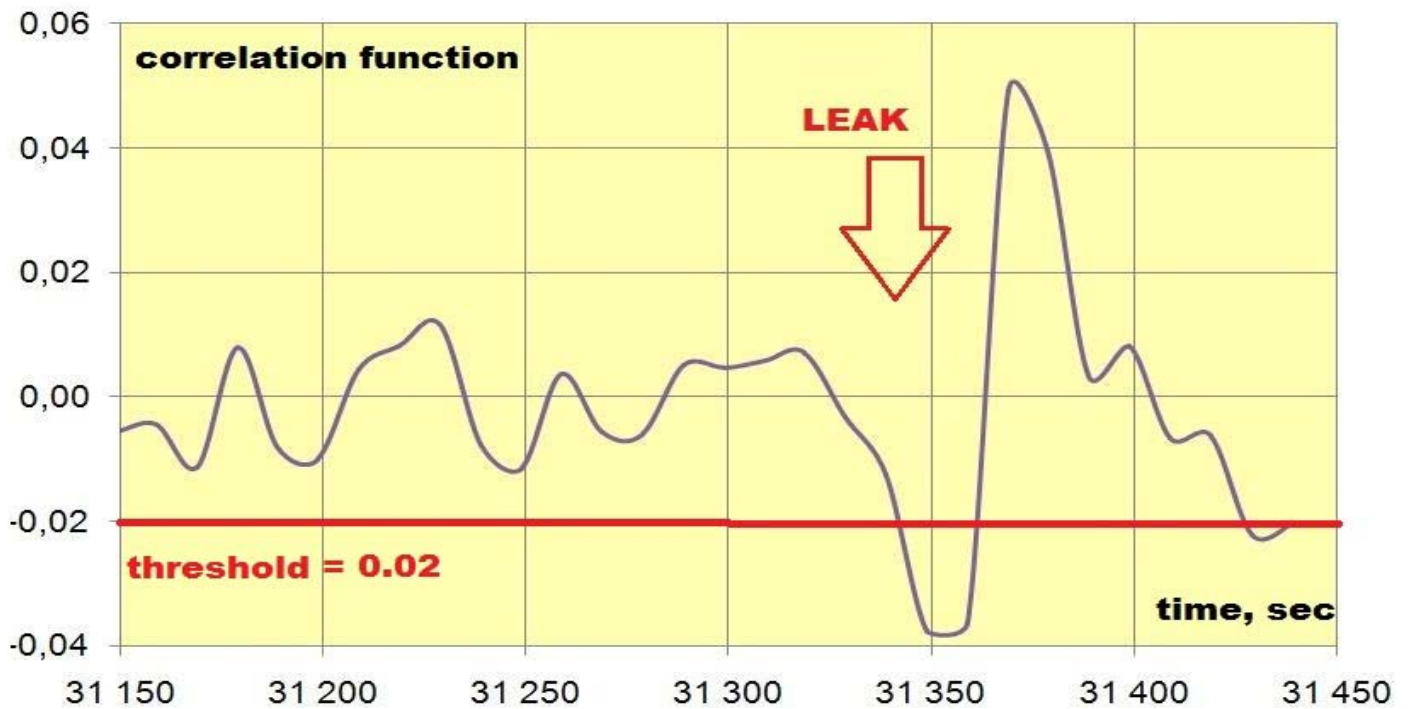


Figure 11 – Oil discharge at 31335. Use of the correlation method on difference between measured and calculated pressure curves.

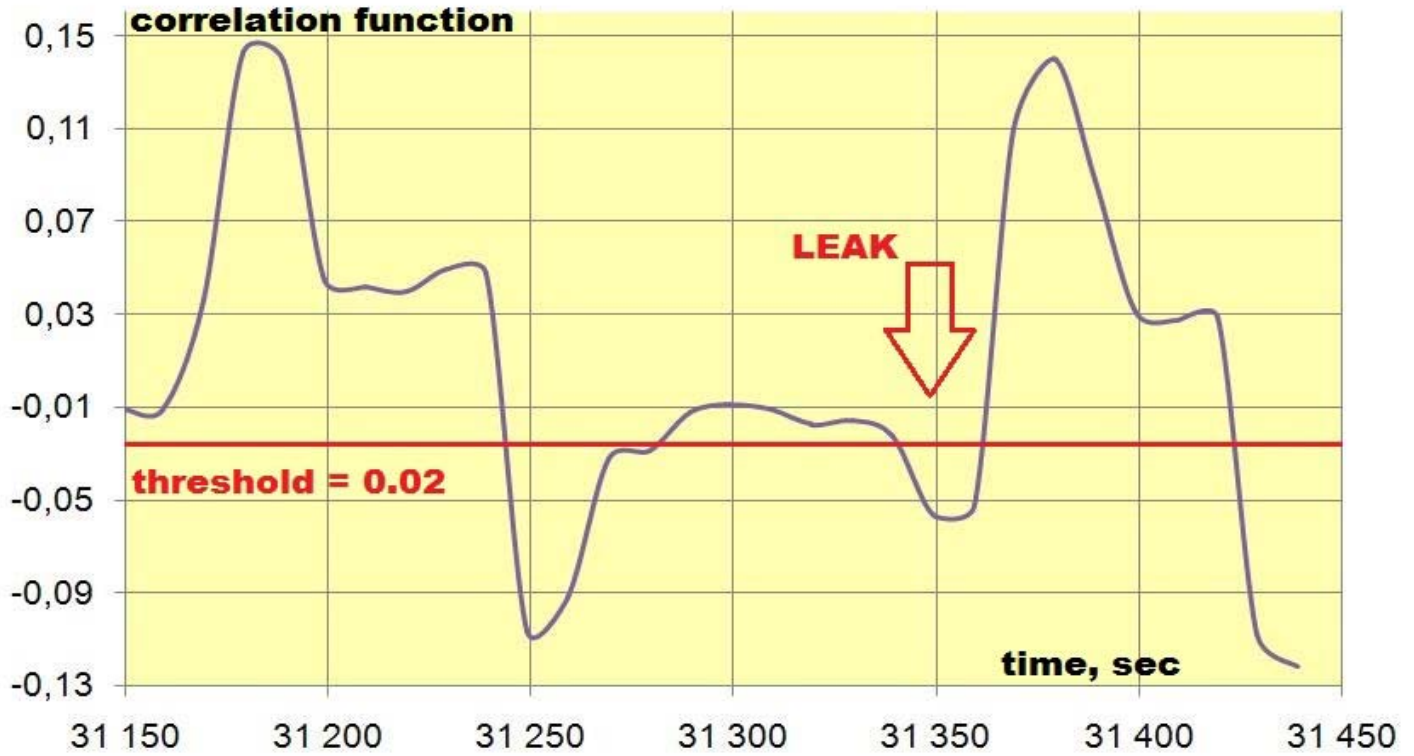


Figure 12 – Oil discharge at 31335. Use of the correlation method on measured pressure curve.